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# Computer Simulation Results and Analysis for a Root-Raised Cosine Filter Design Using Canonical Signed Digits

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# COMPUTER SIMULATION RESULTS AND ANALYSIS FOR A ROOT-RAISED COSINE FILTER DESIGN USING CANONICAL SIGNED DIGITS

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## SUMMARY

Filters implemented using Canonical Signed-Digit (CSD) number representation for the coefficients have been studied for many years as an efficient way to increase the speed and reduce the hardware complexity. A simple and very effective method for designing a CSD filter with a specific set of filter parameters has been investigated and its simulation results are presented here. In order to optimize filter coefficients into the corresponding CSD numbers, the Minimum Mean Square Error (MMSE) criterion is used. Furthermore, an attempt is made to improve frequency response of the CSD filter by allocating an extra nonzero digit for normalized coefficients exceeding one-half. Due to limited filter aperture width in examples presented here, the frequency response of CSD filter as the number of non-zero digits in a CSD code increases is not affected much. A root-raised cosine filter model is employed as a base line for this design approach. Finally, the CSD filter simulation results and hardware complexity comparison with a conventional filter are also shown along with the eye diagrams and Bit-Error-Rate (BER) performance curves.

## I. INTRODUCTION

For a high speed digital filter implementation, Canonical Signed-Digit (CSD) number representation for the coefficients has been studied for many years as a good alternative to expensive parallel processing design or custom application specific integrated circuits (ASIC's) to implement a desired filter for a dedicated channel. This is because when a communication channel is dedicated, the multiplication process in a filter can greatly be simplified by replacing filter coefficients with CSD numbers. The CSD representation of a number is unique and it is also the minimal representation, having no two nonzero digits adjacent to each other. In a radix-2 signed-digit representation, a fractional number can be expressed in the form of

$$x = \sum_{i=1}^K c_i 2^{-n_i}$$

where  $c_i \in \{-1, 0, 1\}$  and  $n_i \in \{0, 1, 2, \dots, K\}$  where  $K$  is the number of nonzero digits. A simple algorithm to convert from a conventional binary representation to CSD representation is given in reference 2. A computer code is written with the MATLAB software package to run the code and generate CSD filter coefficients, and the Alta Group of Cadence's Signal Processing Workstation (SPW) is also used to simulate and analyze the CSD filter responses.

## II. THE DESIGN METHOD

A root-raised cosine finite impulse response (FIR) filter is used as a base line model. The impulse response of the root-raised cosine function is defined by

$$h(t) = \left( \frac{4\beta}{\pi(1-\beta) + 4\beta} \right) \left( \frac{T_s \sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{4\beta t} + \frac{\cos\left(\frac{\pi}{T_s}(1+\beta)t\right) + \frac{4\beta t}{T_s} \sin\left(\frac{\pi}{T_s}(1-\beta)t\right)}{1 - \left(\frac{4\beta t}{T_s}\right)^2} \right), t \neq 0, \pm \frac{T_s}{4\beta}$$

where  $h(0) = 1$  and

$$h\left(\pm \frac{T_s}{4\beta}\right) = \left(\frac{4\beta}{\pi(1-\beta) + 4\beta}\right) \left( \frac{T_s \sin\left(\frac{\pi}{4\beta}(1-\beta)t\right)}{2} + \frac{\pi \cos\left(\frac{\pi}{4\beta}\right) + \pi \sin\left(\frac{\pi}{4\beta}\right)}{4\sqrt{2}} \right)$$

where  $T_s$  is the symbol period in second and  $\beta$  is the rolloff rate.

From this filter, a set of coefficients is sampled and stored in a file. For the all coefficients, the optimal CSD number for each coefficient is searched based upon the MMSE criterion. The distribution of CSD numbers is not uniform. The figures 1 and 2 show nonuniform nature of distribution for all CSD numbers representable with two and three nonzero digits in a 8-bit word length. It also shows that a CSD distribution is more densely populated in smaller numbers (between 0 and 1/2) than bigger numbers (1/2 to 1). Obviously, one with three nonzero digits is better because it has more representable CSD numbers available compared with one with two nonzero digits. The difference between sampled value of a filter and its corresponding CSD-represented numbers are quantization errors, which need to be minimized. The more number of nonzero digits are allowed in a CSD representation, a number can be better approximated and optimized close to a real number. Therefore, number of nonzero digits in CSD representation has a direct impact on quantization errors, and this quantization errors will be translated into undesired frequency responses and Inter-Symbol Interference (ISI). In general, most filters can be implemented with two to four nonzero digits (ref. 2).

Many search techniques for optimum CSD numbers are available. Mixed Integer Linear Programming (MILP) is superior than other local search methods, but heavy computing resources are needed when a number of filter taps is more than around forty (ref. 3). Local search (ref. 4) methods can be very attractive when resources are not available, and it still gives excellent performance even though it is suboptimal to MILP. The technique shown here to optimize the filter coefficients is a local search method in the time domain, using the MMSE with an extra nonzero digit allocation for any coefficient greater than or equal to 1/2. This is a rather simple and time-saving approach, which gives excellent results in coefficient optimization. An extra nonzero digit is permitted because a quantization error from upper half region (from 1/2 to 1) tends to affect filter response more significantly. At the same time, this added nonzero digit will increase hardware complexity of the filter, but much better frequency response results in the end justifies the use of an extra nonzero digit for a small increase in complexity (ref. 2). Other search mechanisms for optimum coefficients in frequency domain using the Fast Fourier Transform (FFT) exist, and for some applications, this may be necessary to meet a filter design specification.

### Flow of Program

A flow chart of the program which converts a fractional decimal number to its closest CSD representation is shown in figure 3. At the start of the program, it takes several input parameters: beta,  $T$ , aperture, length,  $nz$ , pcomp, and delta. Beta is the cosine rolloff rate,  $T$  is the number of samples per symbol, length is the word length,  $nz$  is the number of nonzero digits, pcomp as an optional indicator of having an extra nonzero digit, and delta is the step size in updating a scale factor. A file containing sampled coefficients from the root-raised cosine filter is stored, and each coefficient is recalled sequentially and compared if it is greater than or equal to 1/2. The number of allowed nonzero digits ( $nz$ ) increases by one, while optimizing a decimal number into a CSD representation, if pcomp is set to 1 and a coefficient is greater than or equal to 1/2. Otherwise, it remains the same as the input value to begin with. With newly determined  $nz$  value, the program generates all the possible CSD numbers in a given word length and select the best CSD number that fits a filter coefficient. The Mean squared error (MSE) is calculated and stored to compared with other MSE's from different scale factors. After last coefficient in the file is recalled, the scale factor is changed by a delta amount and the whole process for finding an optimal scale factor is repeated until the scale factor reaches 1/2. The delta, the step size between scale factors, can be set to any desired number and chosen to balance the amount of time spent in searching an optimal scale factor and desired frequency response of filter in the end.

### III. SIMULATION RESULTS

A system model is created in the Alta Group of Cadence's SPW to test the CSD implemented filter response and compare it with conventionally designed filter. As shown in figure 4, two filters in the top (denoted as Filter with Ideal Coefficients) are loaded with sampled coefficients from a root-raised cosine filter. The bottom two filters (denoted as Filter with CSD Coefficients) are filled with CSD-represented numbers. For simulation purposes, a specific set of input parameters is chosen as follows: the number of samples per symbol is 8, word length is 8, aperture is 9, and delta is set to 1/10. The impulse response of the CSD filter (figs. 5, 7, 9) shows that frequency response in the main lobe is as good as one with conventional filter and the side lobes are at least 30 dB down. Eye patterns (figs. 6, 8, 10) are also as good and identical as the other. Furthermore, no increase in sensitivity to timing jitter with CSD filter are observed with the CSD filter. To keep the notation, a consistent notation is used throughout this paper. For example, CSD489847 reads 0.4 rolloff rate, 8 samples per symbol, 9 aperture width, 8 bit word length, four nonzero digits, and 0.7 as the best scaling factor in that order. Figures of impulse responses,  $h(t)$ , for a conventional filter are denoted as for example hof489. It also reads in the same fashion as described earlier that 0.4 rolloff rate, 8 samples per symbol, and 9 aperture width. Simulation results (ref. 5) with different rolloff rates and the number of nonzero digits ( $n_z = 2, 3$  and 4) are collected and a few of those are presented. In general, one nonzero digit in the CSD code is typically required for each 20 dB of stopband attenuation in CSD filter implementation (ref. 2). Frequency responses of a CSD filter with two, three and four nonzero digits are given in figures 11 and 12. Although the side lobe tends to improve as the number of nonzero digit increases, stopband attenuation is not affected significantly. This is due to the filter being aperture-limited. With a greater aperture width, the effects of having more nonzero digits would be clearly translated into its CSD filter response.

Bit error rate (BER) measurements for the cases of CSD289829, CSD289836 and CSD289848 is shown in figure 13. It shows all BER curves perform as good as the theoretical curve. Other BER curves with different set of input parameters show excellent performances as well, and those are found in (ref. 5). It also indicates that the CSD filter with two nonzero digits is the best design choice for implementation. Because with less number of nonzero digits, the same performance level can be achieved as with three or four. The comparison in hardware complexity for those three cases are considered in section IV.

### IV. COMPLEXITY COMPARISON

Three design examples are considered in this section. The number of adders required to implement a CSD coefficient is one less than the number of nonzero digits in the code [2]. All CSD filter coefficients are expressed in the power-of-two. In each table, hardware complexity in CSD filter is compared with conventional filter design case in terms of a total number of adders required to realize the filter. The ratio between the number of adders in the CSD filter and conventional filter is also given. For the case of 8 bit word length, 296 adders ( $8 \times 37$ ) are needed for 37 multipliers in conventional design. Therefore, 368 adders ( $296 + 72$ ) are needed to realize this filter conventionally. With CSD implementation of the filter, only 104 ( $32 + 72$ ) adders are required in the example 1 shown below, and the total of 264 adders is saved through CSD realization. The average number of adders required per filter tap for this CSD implemented filter is 0.86.

### V. CONCLUSIONS

A simple and very effective FIR filter coefficient optimization algorithm has been presented. The optimization process is achieved through searching an optimal scale factor in MMSE sense in conjunction with allocation of an extra nonzero digit to further compensate quantization errors in the CSD representation of filter coefficients. CSD implemented root-raised cosine FIR filter with specific parameters has been simulated and compared with conventional filter design case in frequency response and hardware complexity. In this case study, it is found that CSD filter with two nonzero digits performs as well with lesser hardware as ones with three and four.

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### Example 1

CSD coefficients for 73-tap FIR 289829 filter

$$\begin{array}{lll}
 h(0) = 2^{-6} - 2^{-8} & h(12) = -2^{-5} & h(24) = -2^{-3} - 2^{-5} \\
 h(1) = 2^{-6} + 2^{-8} & h(13) = -2^{-7} & h(25) = -2^{-2} + 2^{-4} \\
 h(2) = 2^{-5} - 2^{-8} & h(14) = 2^{-6} + 2^{-8} & h(26) = -2^{-3} - 2^{-5} \\
 h(3) = 2^{-5} - 2^{-8} & h(15) = 2^{-4} - 2^{-6} & h(27) = -2^{-3} \\
 h(4) = 2^{-5} - 2^{-7} & h(16) = 2^{-4} + 2^{-7} & h(28) = -2^{-4} + 2^{-6} \\
 h(5) = 2^{-6} & h(17) = 2^{-3} - 2^{-5} & h(29) = 2^{-4} + 2^{-7} \\
 h(6) = 0 & h(18) = 2^{-3} - 2^{-5} & h(30) = 2^{-2} - 2^{-5} \\
 h(7) = -2^{-6} & h(19) = 2^{-4} + 2^{-7} & h(31) = 2^{-1} - 2^{-3} \\
 h(8) = -2^{-5} & h(20) = 2^{-5} + 2^{-7} & h(32) = 2^{-1} + 2^{-5} + 2^{-8} \\
 h(9) = -2^{-4} + 2^{-6} & h(21) = -2^{-7} & h(33) = 2^0 - 2^{-3} - 2^{-5} \\
 h(10) = -2^{-4} + 2^{-6} & h(22) = -2^{-4} & h(34) = 2^0 - 2^{-3} + 2^{-5} \\
 h(11) = -2^{-4} + 2^{-6} & h(23) = -2^{-3} + 2^{-7} & h(35) = 2^0 - 2^{-4} \\
 & & h(36) = 2^0 - 2^{-4} + 2^{-6}
 \end{array}$$

TABLE I

	Conventional	CSD implemented	Difference	Ratio
Adders	72	$32 + 72 = 104$	264	1:3.54
Multipliers	37	0	--	----
Delays	72	72	0	1:1

### Example 2

CSD coefficients for 73-tap FIR 289836 filter

$$\begin{array}{lll}
 h(0) = 2^{-7} & h(12) = -2^{-6} - 2^{-8} & h(24) = -2^{-3} + 2^{-6} + 2^{-8} \\
 h(1) = 2^{-6} - 2^{-8} & h(13) = -2^{-7} & h(25) = -2^{-3} + 2^{-8} \\
 h(2) = 2^{-6} & h(14) = 2^{-6} - 2^{-8} & h(26) = -2^{-3} + 2^{-6} - 2^{-8} \\
 h(3) = 2^{-6} + 2^{-8} & h(15) = 2^{-5} & h(27) = -2^{-3} + 2^{-5} + 2^{-7} \\
 h(4) = 2^{-6} & h(16) = 2^{-4} - 2^{-6} & h(28) = -2^{-5} \\
 h(5) = 2^{-7} & h(17) = 2^{-4} - 2^{-8} & h(29) = 2^{-4} - 2^{-6} \\
 h(6) = 0 & h(18) = 2^{-4} - 2^{-8} & h(30) = 2^{-3} + 2^{-6} + 2^{-8} \\
 h(7) = -2^{-6} + 2^{-8} & h(19) = 2^{-4} - 2^{-6} & h(31) = 2^{-2} \\
 h(8) = -2^{-5} + 2^{-7} & h(20) = 2^{-5} - 2^{-8} & h(32) = 2^{-1} - 2^{-3} - 2^{-6} \\
 h(9) = -2^{-5} & h(21) = -2^{-8} & h(33) = 2^{-1} - 2^{-4} + 2^{-6} \\
 h(10) = -2^{-5} - 2^{-8} & h(22) = -2^{-4} + 2^{-6} + 2^{-8} & h(34) = 2^{-1} + 2^{-5} \\
 h(11) = -2^{-5} & h(23) = -2^{-4} - 2^{-6} & h(35) = 2^{-1} + 2^{-4} + 2^{-6} + 2^{-8} \\
 & & h(36) = 2^{-1} + 2^{-3} - 2^{-5} + 2^{-7}
 \end{array}$$

TABLE II

	Conventional	CSD implemented	Difference	Ratio
Adders	72	36 + 72 = 108	260	1:3.4
Multipliers	37	0	--	---
Delays	72	72	0	1:1

### Example 3

CSD coefficients for 73-tap FIR 289848 filter

$$\begin{array}{lll}
 h(0) = 2^{-7} & h(12) = -2^{-5} + 2^{-8} & h(24) = -2^{-3} - 2^{-6} \\
 h(1) = 2^{-6} & h(13) = -2^{-7} & h(25) = -2^{-3} - 2^{-5} - 2^{-8} \\
 h(2) = 2^{-5} - 2^{-7} & h(14) = 2^{-6} & h(26) = -2^{-3} - 2^{-6} + 2^{-8} \\
 h(3) = 2^{-5} - 2^{-7} & h(15) = 2^{-4} - 2^{-6} - 2^{-8} & h(27) = -2^{-3} + 2^{-6} - 2^{-8} \\
 h(4) = 2^{-6} + 2^{-8} & h(16) = 2^{-4} - 2^{-8} & h(28) = -2^{-5} - 2^{-7} \\
 h(5) = 2^{-6} - 2^{-8} & h(17) = 2^{-4} + 2^{-6} & h(29) = 2^{-4} \\
 h(6) = 0 & h(18) = 2^{-4} + 2^{-6} & h(30) = 2^{-2} - 2^{-4} + 2^{-8} \\
 h(7) = -2^{-6} & h(19) = 2^{-4} - 2^{-8} & h(31) = 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} \\
 h(8) = -2^{-5} + 2^{-8} & h(20) = 2^{-5} + 2^{-8} & h(32) = 2^{-1} - 2^{-5} + 2^{-7} \\
 h(9) = -2^{-5} - 2^{-7} & h(21) = -2^{-7} & h(33) = 2^{-1} + 2^{-3} - 2^{-6} - 2^{-8} \\
 h(10) = -2^{-4} + 2^{-6} + 2^{-8} & h(22) = -2^{-4} + 2^{-7} & h(34) = 2^0 - 2^{-3} - 2^{-6} - 2^{-8} \\
 h(11) = -2^{-5} - 2^{-7} & h(23) = -2^{-3} + 2^{-5} - 2^{-7} & h(35) = 2^0 - 2^{-3} + 2^{-6} - 2^{-9} \\
 & & h(36) = 2^0 - 2^{-3} + 2^{-5} - 2^{-7} + 2^{-9}
 \end{array}$$

TABLE III

	Conventional	CSD implemented	Difference	Ratio
Adders	72	48 + 72 = 120	248	1:3.1
Multipliers	37	0	--	---
Delays	72	72	0	1:1

CSD numbers distribution:  $n = 8, nz = 2$

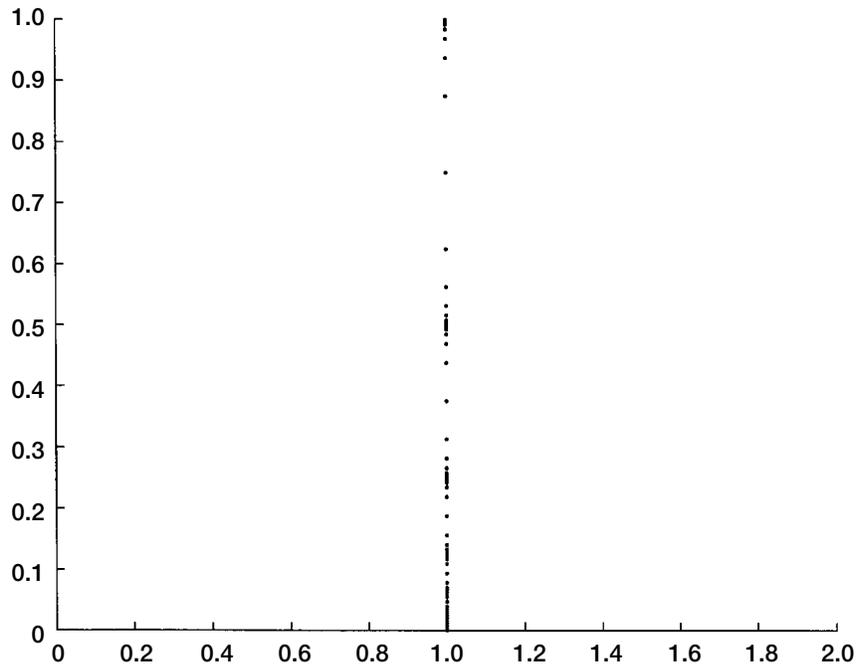


Figure 1.

CSD numbers distribution:  $n = 8, nz = 3$

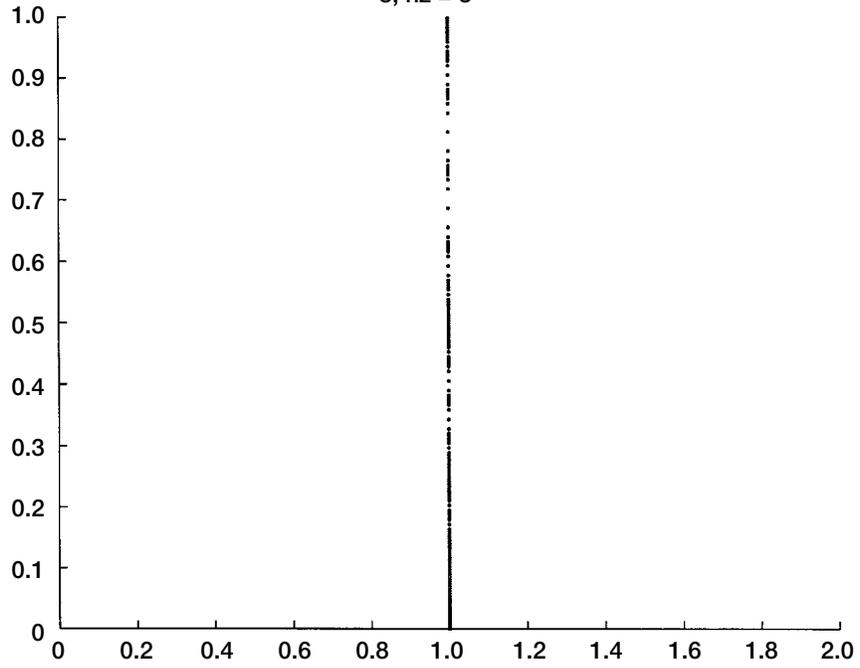


Figure 2.

Flow chart for csdmap.m

pcomp: precompensation  
 nz: number of nonzeros  
 sw: switch

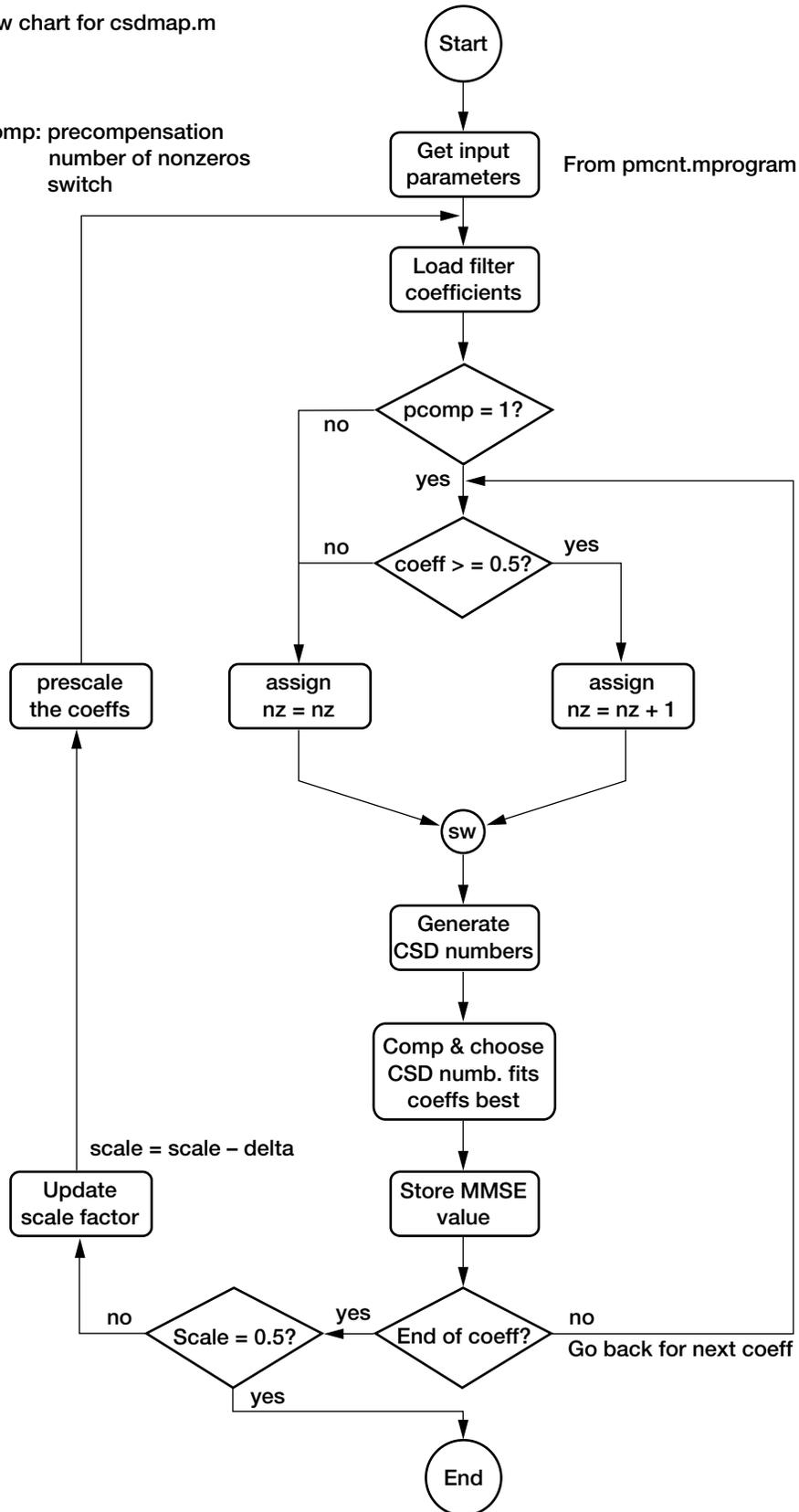


Figure 3.

CSD filter ber measurement system

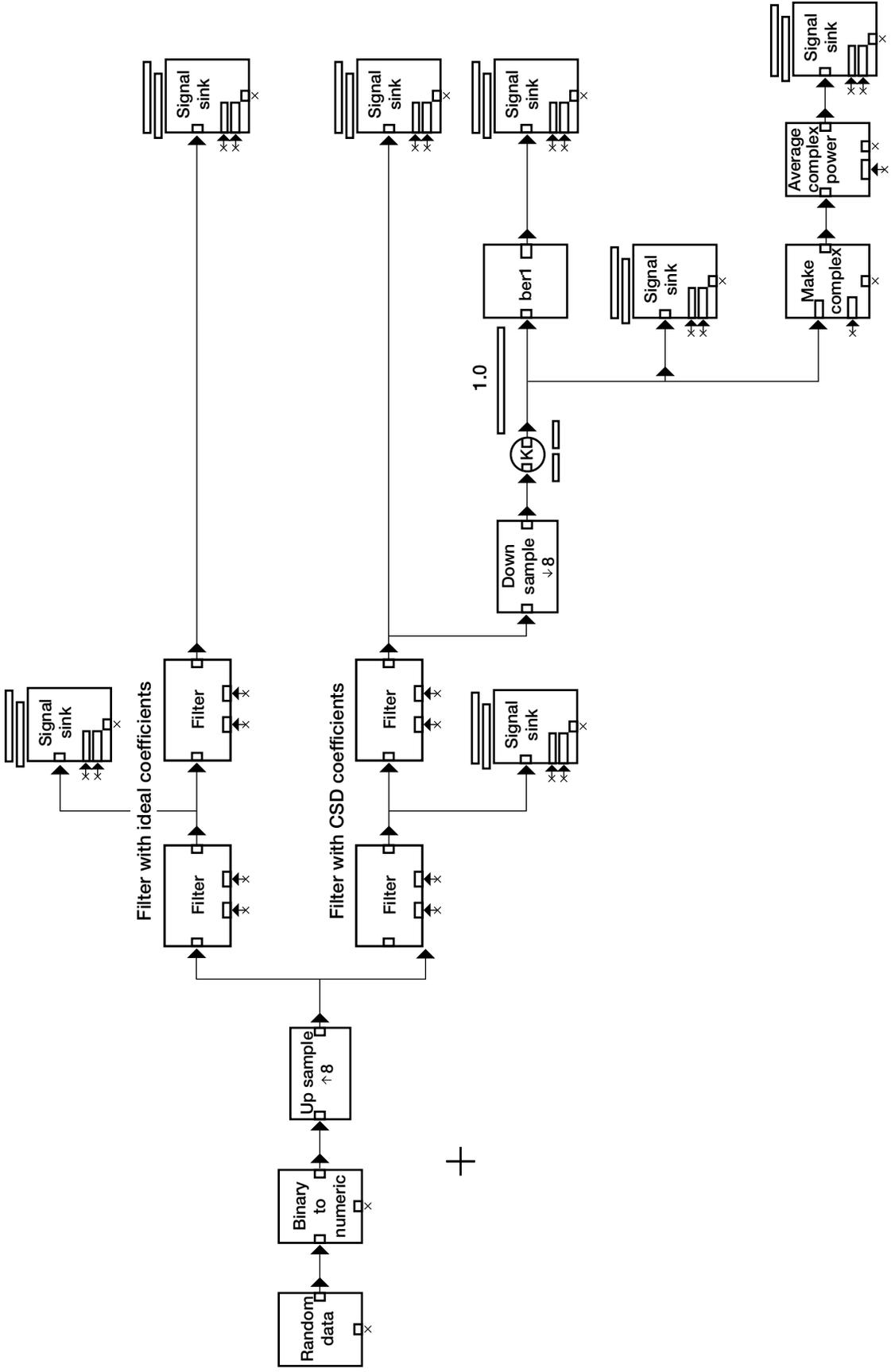


Figure 4.

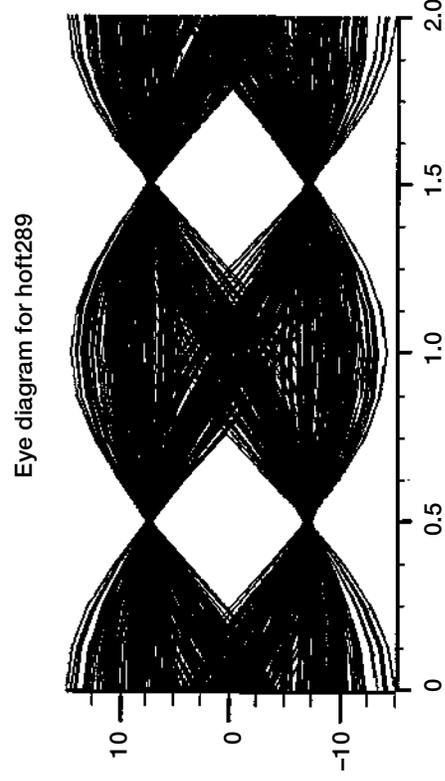
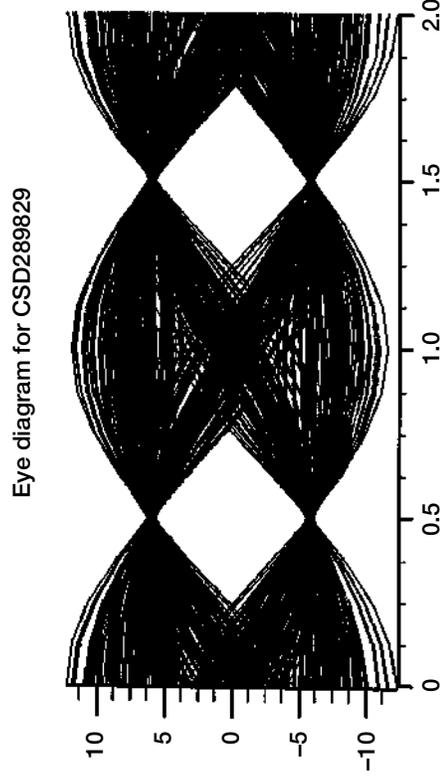
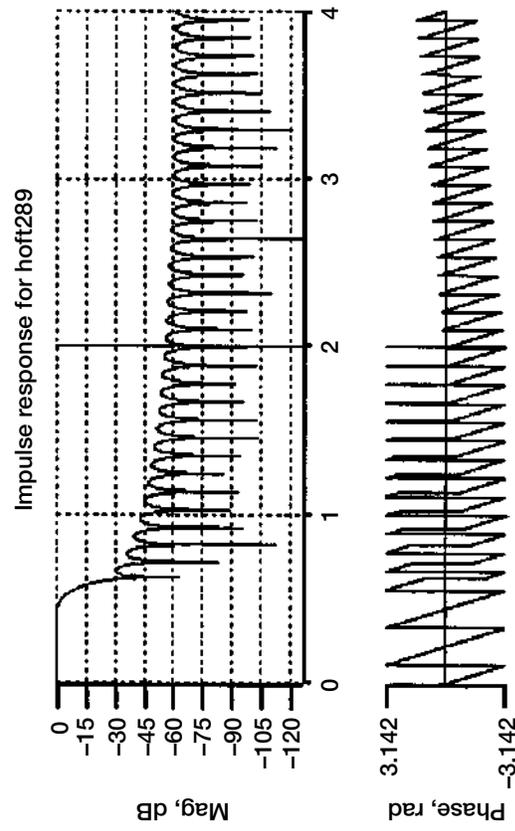
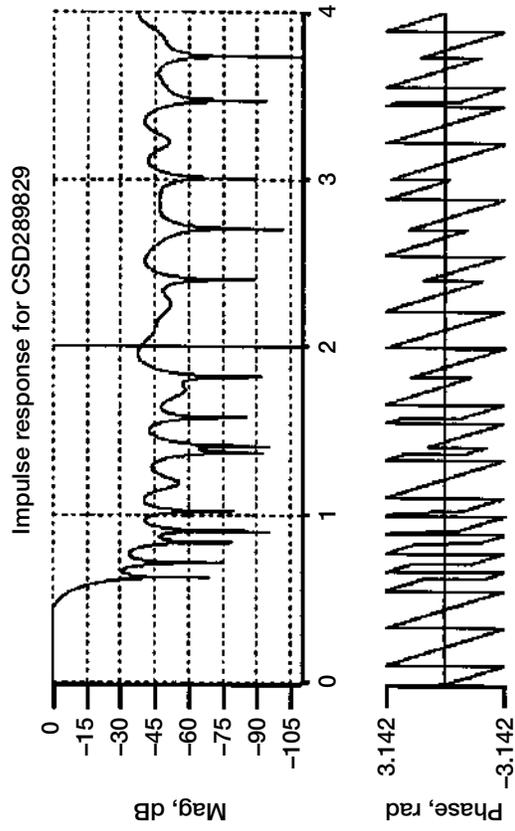
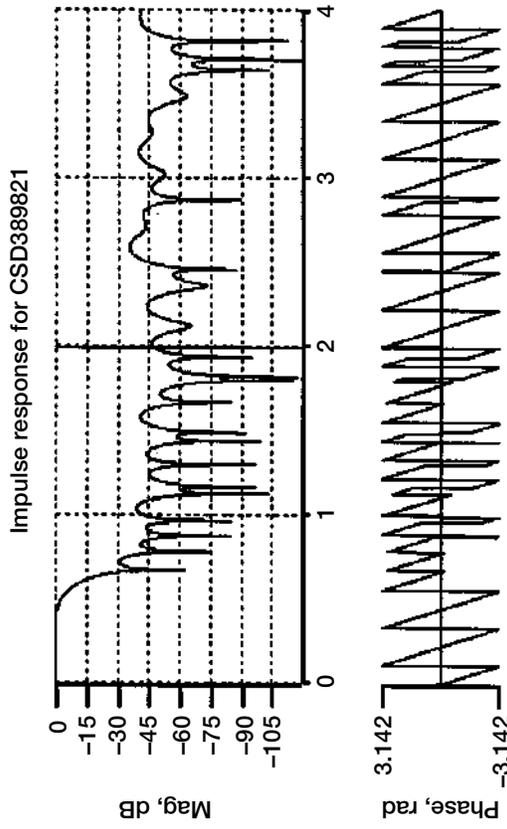


Figure 6.

Figure 5.



10

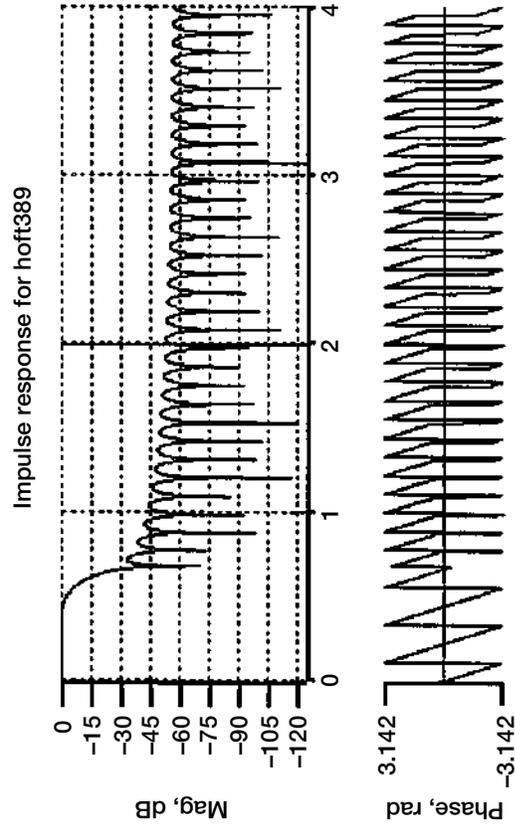


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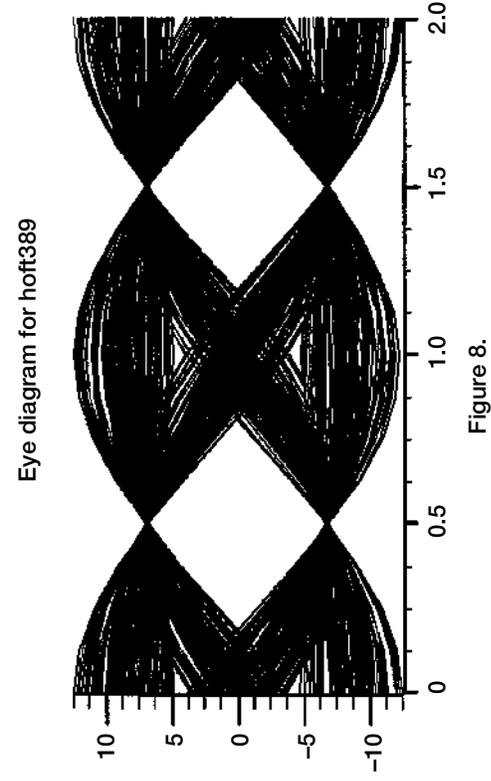
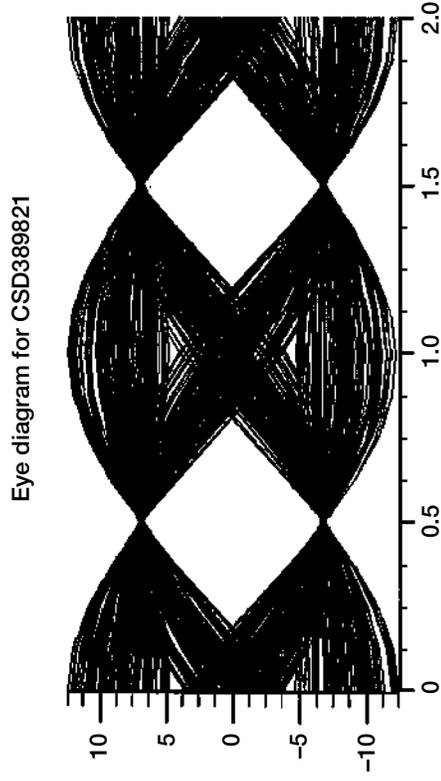


Figure 8.

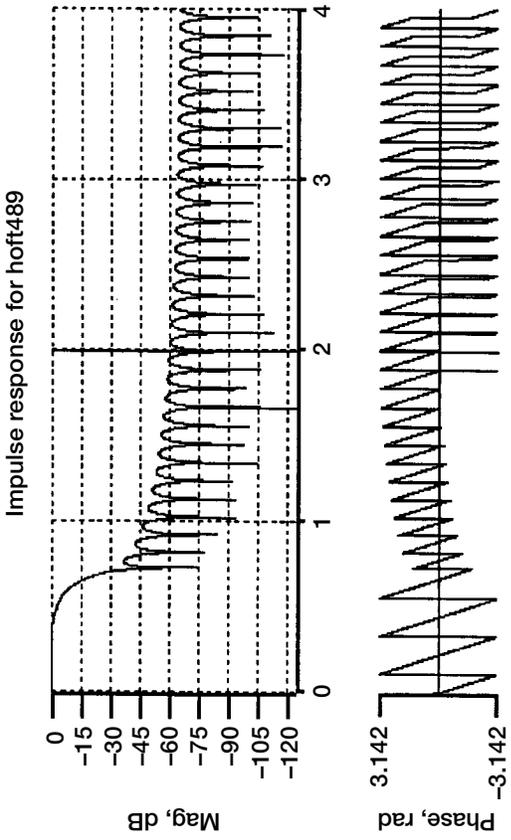
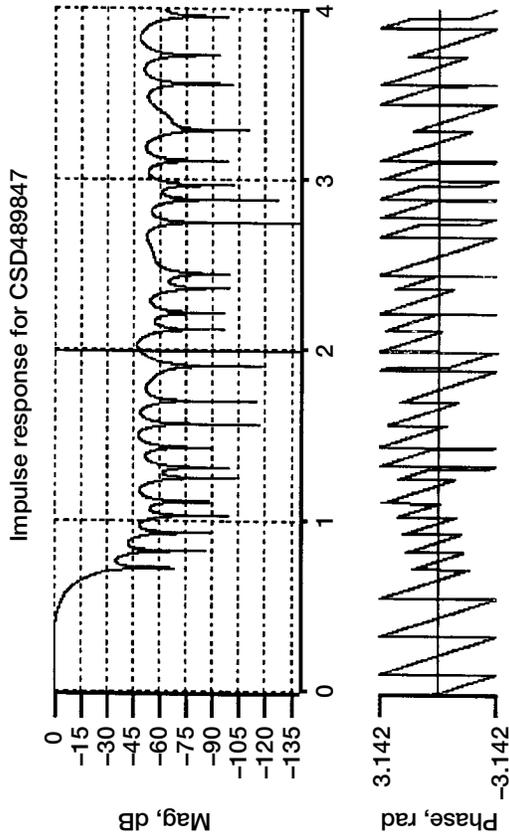


Figure 9.

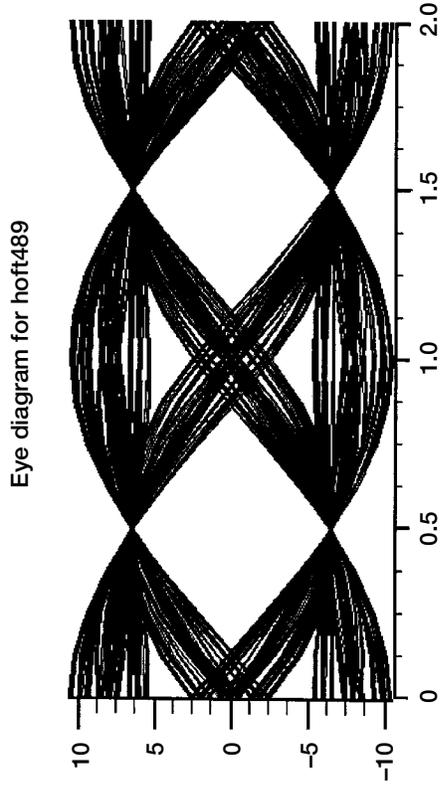
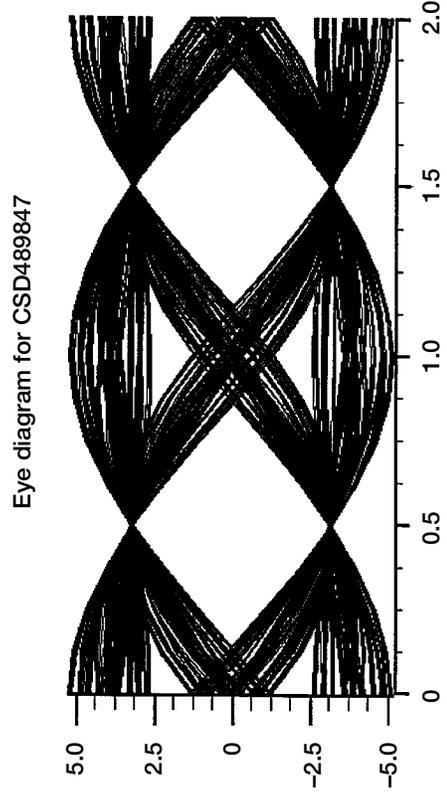


Figure 10.

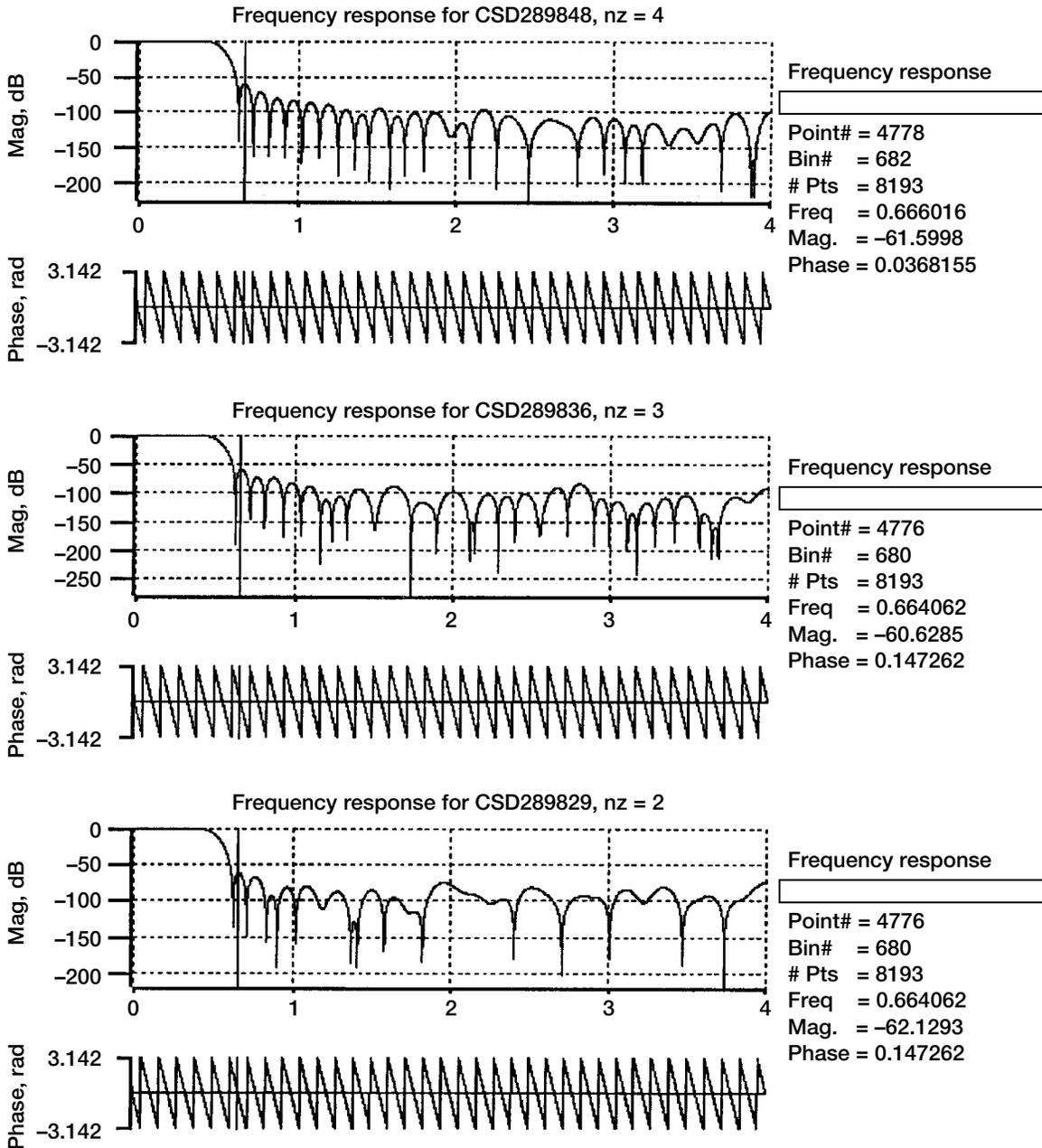


Figure 11.

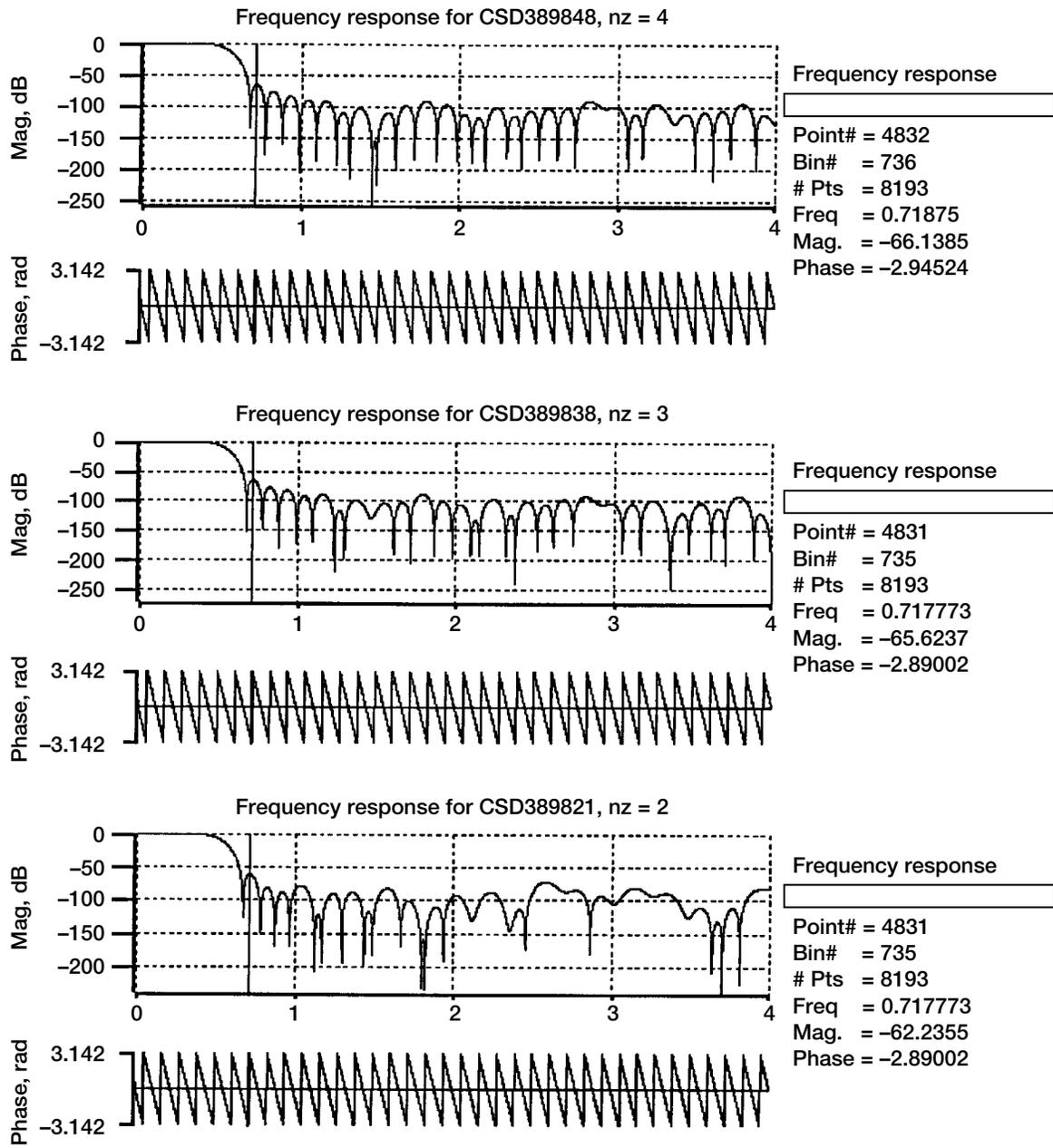


Figure 12.

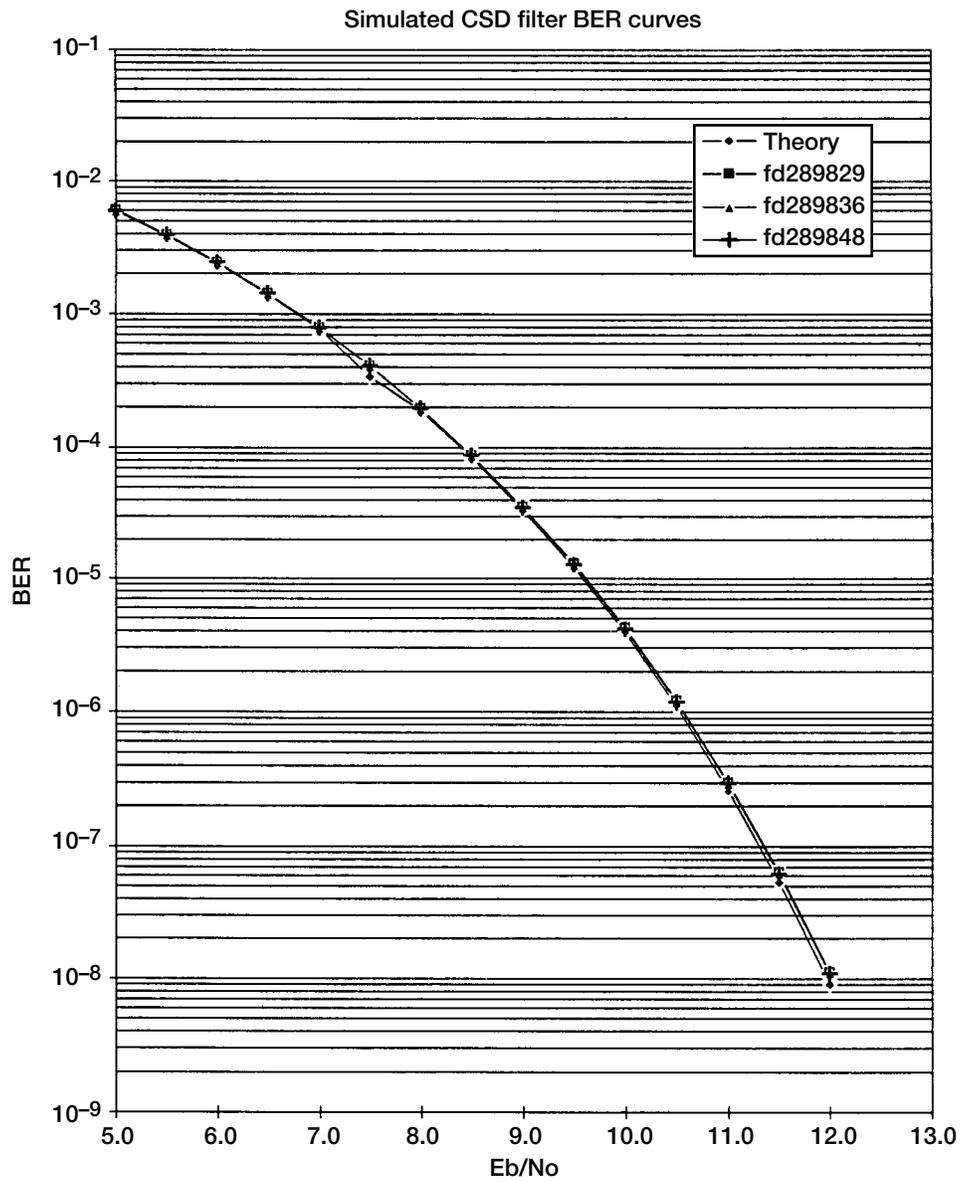


Figure 13.

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